SOME PROPERTIES OF MEMORYLESS MULTITERMINAL INTERFERENCE CHANNELS

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Abstract

This paper examines some properties of memoryless multiterminal interference channels. A general formula for the capacity of such channels is presented. However this formula does not easily lend itself to computation. Motivated by the fact that for a single user memoryless channel (1/N)Sup[I(X;Y)] is independent of N (where **X** and **Y** are N-vectors and the suprema is taken over all probability distributions on **X**), we are investigating the possibility that the general capacity formula has an analogous simplification. A specific example of a Gaussian Interference channel with two independent users is considered. It is shown that certain probability distributions for N=2 achieve points on the boundary of the capacity region. We hope that some form of this result will generalize to other multiterminal communication channels, and possibly give some insight into the nature of the channel capacity in terms of a single-letter description. Presently, the capacity region of such channels is, in general, unknown.

Summary

We begin by considering the channel shown in Figure 1 below. It is desired that two independent users each communicate simultaneously with a single receiver. We are interested in the capacity region, i.e., the set of all rates (R_1,R_2) that are simultaneously achievable with arbitrarily small probability of error.

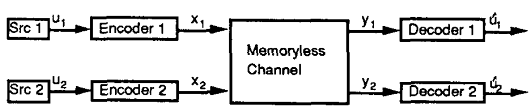

Figure 1. Two independent users each communicate with a single receiver.

Theorem 1. The capacity region of the channel shown in Figure 1 is:

$$C = \text{Closure}\left\{\bigcup_{N=1}^{\infty}\bigcup_{P} R\left[\frac{1}{N}I(\mathbf{X}_1;\mathbf{Y}_1),\ \frac{1}{N}I(\mathbf{X}_2;\mathbf{Y}_2)\right]\right\}$$

where $\mathbf{X}_1, \mathbf{X}_2, \mathbf{Y}_1, \mathbf{Y}_2$ are N-vectors
P is the set of all independent probability distributions on $\mathbf{X}_1$ and $\mathbf{X}_2$
and $R[x, y] = \{ (R_1, R_2):\ 0 \le R_1 \le x \text{ and } 0 \le R_2 \le y \}$.

Theorem 1 can be generalized to treat multiple transmitters and receivers. However, such a formula for capacity does not lend itself to computation because it is an infinite union of convex regions. Motivated by the fact that for a single-user memoryless channel (1/N)Sup[I(**X**;**Y**)] is independent of N (where **X** and **Y** are N-vectors and the suprema is taken over all probability distributions on **X**), we are investigating the possibility that this general capacity formula has an analogous simplification.

Example. Consider a Gaussian Interference Channel with $y_1 = x_1 + x_2 + z_1$. and $y_2 = x_1 + x_2 + z_2$.

where x_1, x_2 independent, $E\{x_1^2\} \le 1$, $E\{x_2^2\} \le 1$,
$z_1, z_2 \sim N(0, 1)$ independent.

Since y_1 and y_2 are statistically equivalent, the capacity region of this channel can be determined[2] and is shown Figure 2. By replacing the infinite union in Theorem 1 by a a union over finitely many values of N, we obtain a subset of **C**. Figure 2 shows some achievable points for N=1 and N=2. In particular, we see that for N=2 a point on the boundary of **C** is attained. Thus we can say that the expression for channel capacity given in Theorem 1 does not approach the capacity "continuously" as N tends to infinity, because this point on the boundary can be achieved for N=2. We hope that some form of this result will generalize to other multiterminal communication channels, and possibly give some insight into the nature of the channel capacity in terms of a single-letter description.

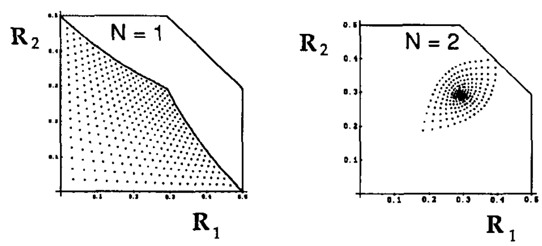

Figure 2. Some achievable points in **C**.

References

[1] T. M. Cover, "Some Advances In Broadcast Channels," in *Advances in Communication Systems*, vol. 4. New York: Academic Press, 1975, pp. 229-260.

[2] Aydano Carleial, "Interference Channels," *in IEEE Transactions on Information Theory,* Vol. IT-24, No. 1, January 1978, pp. 60-70.

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